Observations of wave-induced pore-pressure gradients and bed level response on a surf zone sandbar

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Physical mechanisms driving sandbar migration

Offshore migrations

Large waves break on the sand bar

Undertow currents directed offshore

Bar moves offshore

(adapted from Hoefel & Elgar 2003)
Physical mechanisms driving sandbar migration

Onshore migrations

Calmer wave conditions, weak currents

Asymmetric wave shapes

Bar moves onshore

\[ As = \frac{\frac{\partial u}{\partial t^3}}{\left(\frac{\partial u}{\partial t^2}\right)^{3/2}} \]
Shields parameter

\[ \theta = \frac{\tau_b}{(\rho_s - \rho_w)g a_{50}} \]

Sheet flow

\[ \theta = 0.8 \]
Considering pressures within the sediment bed: horizontal forcing

Shields parameter
\[ \theta = \frac{\tau_b}{(\rho_s - \rho_w)g d_{50}} \]

Sleath parameter
\[ S = \frac{\partial p/\partial x}{(\rho_s - \rho_w)g} \]
Considering pressures within the sediment bed: horizontal forcing

Shields parameter

$$\theta = \frac{\tau_b}{(\rho_s - \rho_w)g d_{50}}$$

Sleath parameter

$$S = \frac{\partial p/\partial x}{(\rho_s - \rho_w)g}$$

Bed failure ≈ “plug flow”

$$S_f = 0.1 \ (Foster \ et \ al. \ 2006)$$

(adapted from Sleath 1999)
\[-\frac{1}{\rho_f} \frac{\partial p}{\partial x} = \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + w \frac{\partial u}{\partial z}\]

\[As = \left(\frac{\partial u/\partial t}{\partial u/\partial t^2}\right)^{3/2}\]

Shields parameter
\[\theta = \frac{\tau_b}{(\rho_s - \rho_w)g d_{50}}\]

Sleath parameter
\[S = \frac{\partial p/\partial x}{(\rho_s - \rho_w)g}\]
Can we develop a more holistic understanding of the forces within the bed?

Oscillatory flow

Wave propagation

spatial and temporal gradients in the flow
Long Wave Flume
Length: 104m
Width: 3.7m
Depth: 4.6m

Piston-type wavemaker:
- Regular, Irregular, Tsunami
- Max wave: 1.7m @ 5 seconds
Hybrid profile construction
- limits large-scale bathymetric-hydrodynamic feedbacks
- isolates small-scale bed response to varying wave forcing

d_{50} = 0.17 \text{ mm}
d_{50} = 0.27 \text{ mm}
Hybrid profile construction

- limits large-scale bathymetric-hydrodynamic feedbacks
- isolates small-scale bed response to varying wave forcing
Pore-Pressure Transducer Array

- Buried within sediment bed
- Druck PDCR 81s, sampling at 100 Hz
- Finite differencing
Pore-Pressure Transducer Array

\[ P_{xi} = -2P_i - 3P_i + 6P_{i+1} - P_{i+2} \]
\[ 6\Delta x = -2x^2 - 3x^3 + 6x^4 - x^5 \]

\[ P_{zi} = -3P_i + 4P_{i+1} - P_{i+2} \]
\[ 2\Delta z = -3x_3 + 4z_1 - z_2 \]
Conductivity Concentration Profiler

- Buried within sediment bed
- Instantaneous bed levels
- 32 mm window, sampling at 100z
62 trials consisting of 26 different wave conditions

\begin{align*}
H_{\text{bar}} & \quad H_i & \quad \text{Group} \\
\bullet & \quad \Delta & \quad 1 \ (3.5 \text{ s}) \\
\bullet & \quad \triangle & \quad 2 \ (5.0 \text{ s}) \\
\bullet & \quad \triangle & \quad 3 \ (7.0 \text{ s}) \\
\bullet & \quad \triangle & \quad 4 \ (9.0 \text{ s})
\end{align*}

\[ 2a/h = 0.78 \]
Ensemble averaging to create pressure gradient vector $\vec{\nabla}p$
Rotations of $\vec{V}p$ with each wave cycle

Group 1

$S_{plug} = 0.1$

Group 2

Group 3

Group 4

shore

time (s)
Steep, short period waves can produce the same onshore $S$ as less steep long period waves with larger $A_s$ values.
\[-\frac{1}{\rho_f} \frac{\partial p}{\partial x} = \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + w \frac{\partial u}{\partial z}\]

Advevtive terms account for 14% of the horizontal pore pressure gradient.
Shields parameter
\[ \theta = \frac{\tau_b}{(\rho_s - \rho_w)gd_{50}} \]

Sleath parameter
\[ S = \frac{\partial p / \partial x}{(\rho_s - \rho_w)g} \]
Shields parameter
$$\theta = \frac{\tau_b}{(\rho_s-\rho_w)gd_{50}}$$

Sleath parameter
$$S = \frac{\partial p/\partial x}{(\rho_s-\rho_w)g}$$

Erosion depth is function of both shear and pressure gradient

4 mm ≈ 24 grain diameters
Bed failures coincident with spikes in onshore-directed $S$
$S_f$ is not universal, and has no clear dependence on $As$.

Initiation of erosion likely depends on a combination of shear and pressure gradients competing to secure or destabilize sediment skeleton.

(Foster et al. 2006; Frank et al. 2015; Cheng et al. 2016)
• Obtained a pore pressure gradient, $\vec{V}p$, within the sediment bed beneath complicated hydrodynamics over a surf zone sandbar

$$\frac{d\rho}{dx} - \frac{\tau_s}{h_b} > KC_*(\rho_s - \rho)g + K \frac{dp}{dz}$$

(Sleath [1999], Foster et. al. [2006])

• Magnitude of onshore-directed Sleath not directly related to As

• Rapid drops in bed level were coincident with spikes in pore pressure gradient, but not at a universal critical failure point
## Natural Hazards Engineering Research Infrastructure (NHERI) program

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\[-\frac{1}{\rho_f} \frac{\partial p}{\partial x} = \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + w \frac{\partial u}{\partial z}\]
Deriving gradients by finite difference approximations

\[ P_{i-1} = P_i - \Delta x P_{xi} + \frac{1}{2} \Delta x^2 P_{xxi} - \frac{1}{6} \Delta x^3 P_{xxx} + O(\Delta x^4) \]
\[ P_i = P_i \]
\[ P_{i+1} = P_i + \Delta x P_{xi} + \frac{1}{2} \Delta x^2 P_{xxi} + \frac{1}{6} \Delta x^3 P_{xxx} + O(\Delta x^4) \]
\[ P_{i+2} = P_i + (2\Delta x) P_{xi} + \frac{1}{2} (2\Delta x)^2 P_{xxi} + \frac{1}{6} (2\Delta x)^3 P_{xxx} + O(\Delta x^4) \]

\[ P_{xi} \approx \frac{1}{\Delta x} \sum_j \alpha_j P_j \]
\[ = (\alpha_{i-1} + \alpha_i + \alpha_{i+1} + \alpha_i) \frac{1}{\Delta x} P_i \]
\[ = (-\alpha_{i-1} + \alpha_{i+1} + 2\alpha_i) P_{xi} \]
\[ = \left( \frac{1}{2} \alpha_{i-1} + \frac{1}{2} \alpha_{i+1} + \frac{4}{6} \alpha_i \right) \Delta x P_{xxi} \]
\[ = \left( -\frac{1}{6} \alpha_{i-1} + \frac{1}{6} \alpha_{i+1} + \frac{8}{6} \alpha_i \right) \Delta x^2 P_{xxx} \]

**Horizontal**
\[ \frac{\partial (p/\gamma)}{\partial x} = P_{xi} = \frac{-2P_{i-1} - 3P_i + 6P_{i+1} - P_{i+2}}{6\Delta x} = \frac{-2x_2 - 3x_3 + 6x_4 - x_5}{6\Delta x} \]

**Vertical**
\[ \frac{\partial (p/\gamma)}{\partial z} = P_{zi} = \frac{-3P_i + 4P_{i+1} - P_{i+2}}{2\Delta z} = \frac{-3x_3 + 4z_1 - z_2}{2\Delta z} \]