Substrate effects from force chain dynamics in dense granular flows

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[1] Granular materials are composed of solid, discrete particles and exhibit mechanical properties that range from fluid to solid behavior. Some of the complexity exhibited by granular systems arises due to the long-range order that develops due to particle-particle contact. Inter-particle forces in granular materials often form a distributive network of filamentary force-accommodating chains (i.e., force chains), such that a fraction of the total number of particles accommodates the majority of the forces in the system. The force chain network inherent to a system composed of granular materials controls the macroscopic behavior of the granular material. Force transmission by these filamentary chains is focused (or localized) to the grain scale at boundaries such as the granular flow substrate. This investigation addresses the effects of force localization on the substrate by dynamic force chain processes and the implications for bed entrainment in dense, unconfined, two-dimensional, gravity-driven granular flows. Our experimental system employs photoelastic techniques and provides an avenue for quantitative force analysis via image processing and provides a data set that can be used validate discrete element modeling approaches. We show that force chains cause extreme bed-force localization in dynamic granular systems, and that these localized forces can propagate extensively into the substrate, even ahead of the flow front.


1. Introduction

[2] Granular materials are composed of solid, discrete particles that dissipate energy when the constitutive particles interact — for example, through friction or collision [Behringer et al., 2008]. Granular materials exhibit mechanical properties that range from fluid to solid behavior and granular materials can have varied rheological response when in an excited state, that is when the materials are vibrated or allowed to flow [Frey and Church, 2011; Jaeger et al., 1996; Sun et al., 2010]. Rheology of granular materials is principal to a suite of geophysical processes, including dry ravel, sand dune migration, the motion of landslides, debris flows, pyroclastic flows, avalanches, and fault systems [Daniels and Hayman, 2008; Dufek and Bergantz, 2007; Furbish et al., 2008; Iverson et al., 2011; LaBerge et al., 2006; Majmudar and Behringer, 2005].

[3] Recent experimental studies have highlighted that inter-particle forces in granular materials often form a distributive network of filamentary force-accommodating chains (i.e., force chains) as opposed to having an isotropic distribution of forces, such that a fraction of the total number of particles accommodate the majority of body forces and externally applied forces [Majmudar and Behringer, 2005; Geng et al., 2003; Behringer et al., 2008; Sun et al., 2010]. The ability of a granular system to resist deformation is a function of the force chain network formed within the granular system [Furbish et al., 2008; Sun et al., 2010]. The responses of these chain networks to applied stresses ultimately define the material’s macroscopic character.

[4] Force chains, because they are responsible for anisotropic force distributions in granular media, could carry significant implications for granular processes and have been the subject of recent theoretical work [Furbish et al., 2008; Campbell, 2006]. Furbish et al. [2008] scaled the production and destruction of force chains with the flow shear rate, and allowed additional destruction caused by acoustic vibrations or grain collisions. Campbell [2006] defined granular flows exhibiting force chains as elastic flows. Formulations in the elastic flow theory were derived using a constant volume constraint. Localized forces via force chains have been well documented for confined, static and shearing granular systems [Behringer et al., 2008; Majmudar and Behringer, 2005; Majmudar et al., 2007; Muthuswamy and Tordesillas, 2006]. However, it remains unclear if free surface (unconfined) flows, common in Nature, follow the same behavior. The primary questions we want to address with this work are as follows: (1) Do the localized forces characteristic of confined granular systems translate to unconfined dynamic systems, and if so are they important for bed forces exerted on the substrates of

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unconfined gravity-driven granular flows? (2) If force chains are present at the flow boundaries of dynamic granular systems, do the effects from force chains in unconfined gravity-driven granular flows carry significant implications for erosion and entrainment of the substrate? Such modifications in erosion and entrainment may have implications for enhanced mobility of gravity-driven granular flows [Berger et al., 2011; Calder et al., 2000; Crosta et al., 2009; Mangeney et al., 2007; Iverson et al., 2011; Dufek et al., 2009]. The entrainment of substrate material by the flowing mass could potentially increase or decrease flow mobility depending on flow dynamics and the physical character of the substrate [Berger et al., 2011; Iverson, 1997; Iverson et al., 2011; Mangeney et al., 2010].

Our experiments generated two-dimensional, monodisperse, gravity-driven granular flows. Bed force data were acquired using a modified photoelastic technique, and we have focused on a simple system to improve our ability to analyze this complex problem. However, with these necessary simplifications we offer the caveat that our experiments and results are not intended to be interpreted as a direct analogue for naturally occurring events. Naturally occurring systems involve complexities that are not addressed by our work, for example, polydispersity, irregular terrain, pore fluids, and three-dimensional structure; and it is unclear how our results carry over into these more complex regimes. Our experiments represent an end-member case of dry granular flow systems. Despite these limitations, our approach allows us to investigate a poorly understood mechanism that contributes to bed physics in natural processes involving granular materials.

2. Experimental Techniques

The photoelastic method utilizes the properties of transparent materials, such as glass or certain polymers that become birefringent when stressed. When viewed in a field of circularly or plane-polarized light, one can observe the transmission of the internal stresses caused by forces acting on the material boundaries, which is the basis of the photoelastic method [Majmudar and Behringer, 2005]. Birefringence has been used previously, primarily in structural engineering models, to observe elastic stress distributions in small-scale models of structures in polarized light such as dams, bridges, and mechanical components [Behringer et al., 2008]. The same technique can be used to observe the development and evolution of force chains within a sample of a granular material made of transparent particles.

This photoelastic technique provides an avenue for quantitative analysis of force chain networks, because the intensity of transmitted light is proportional to the boundary forces (Figure 1) [Behringer et al., 2008; Majmudar and Behringer, 2005]. Figure 2 shows a schematic of the light/lens setup used for our photoelastic experiments, and Figure 3 illustrates the shearing motion of the 2D flows and provides a snapshot image of a transient force chain network. Shearing and compression experiments conducted by Behringer et al. [2008], Majmudar and Behringer [2005], and Sun et al. [2010] indicate mesoscale (which falls between particle-scale and macro-scale lengths) force propagation in filamentary force chain networks, in which a fraction of the total number of particles carries the majority of the force [Geng et al., 2003; Sun et al., 2010]. While force chains have been observed as mesoscale features in length, it is important to note that termination of the force propagation (especially at system boundaries) often occurs over the contact of a single particle (see Figures 1 and 3). The range of birefringence shown by disks in Figure 3 illustrates that a fraction of the disks indeed accommodate system forces, but also that different levels of participation by the disks in the force chain network exist. For example, disks that are primary members of a force chain exhibit strong birefringence while spectator disks show no birefringence; and secondary force chain members can be identified by minor birefringence. The levels of participation by force chain members motivate a threshold scheme for analysis, which is described in the calibration section.
2.1. Components

To use the photoelastic technique and implement it into our work, we constructed an apparatus to house the material constituting our granular system. Internal structuring for the apparatus consisted of: the release gate, ramp, run-out bed, erodible bed section, retaining wall, and spacers. These parts were constructed from \(3.18 \times 10^{-3} \text{ m}\) thickness cell cast clear Plexiglas sheets, and \(7.62 \times 10^{-5} \text{ m}\) thickness clear polyester film sheets by Dupont. Clear glass panes measuring \(0.91 \text{ m} \times 0.61 \text{ m} \times 2.38 \times 10^{-3} \text{ m}\) housed the internal acrylic members. Delivery and collection hoppers, used for delivery and removal of the granular media from the apparatus, were also constructed from the Plexiglas material. The collection hopper (Figure 4) was attached prior to the onset of the flows, and added 0.30 m to the available horizontal run-out. Two ramp configurations were employed during the experiments: rigid and erodible. The rigid configuration was a straight, rigid bed positioned at a desired inclination. The erodible configuration was similar to the rigid, but included a 0.12 m long \(\times \) 0.04 m deep rectangular section cut out of the bed, initiated 0.43 m from the upper end of the ramp. Photoelastic disks occupied this cutout area to replicate erodible substrate conditions.

The granular material we chose for our experiments consisted of \(0.006 \text{ m}\) diameter, \(1180 \text{ kg/m}^3\) density disks made from Vishay Precision Group’s PSM-4 PhotoStress model material. Because of its low elastic modulus and uniform sensitivity character, this material was deemed the most suitable for our experiments. The disks were lightly coated with flour to minimize inter-disk adhesion and disk-wall friction, and from 2100 to 2400 disks were used during each experimental run. We recorded free-fall times from disks inside and outside the apparatus to test for frictional effects due to particle-wall interactions. Free-falling particle velocities inside and outside the apparatus revealed no determinable difference. We consider wall effects negligible on the basis that estimated particle-wall forces are very small relative to the measured particle-bed forces.

Figure 2. A schematic diagram of the plane polariscope setup used for our photoelastic experimentation. Not depicted here is a collection hopper (shown in Figure 4), which was attached to the end of the apparatus. Images depicted here are not drawn to scale.

![Figure 2](image1.png)

Figure 3. Snapshot of experimental flow illustrated with inclined \(x\)-\(z\) coordinate system and shear velocity (\(u\)) diagram. The optical activity of the photoelastic disks makes primary force-chain participants, secondary participants, and spectators easily identifiable.
The contact stiffness for the photoelastic disks can be calculated as a function of deformation using the Hertzian contact model from the material’s Young’s modulus, Poisson ratio, and the disk radius [Coste and Giles, 1999; Campbell, 2006]. Based on the magnitude of particle deformation observed in our experiments we calculated contact stiffness for the experimental disks to fall within a range of 1350 to 2600 N/m. These are low stiffness values relative to those expected for rock or sand, however we believe the expected differences in contact timescales (stiffer particles having shorter contact durations) are not limiting to chain formation and thus force propagation. This argument will be revisited in the discussion.

The coefficients of restitution for particle-particle and particle-wall contacts were measured as 0.355 ± 0.121 and 0.473 ± 0.091, respectively. Error values represent one standard deviation for the cumulative data about the mean. These values originate from trials of free falling particles impacting the designated material while being recorded by a high-speed camera. Particle velocities were calculated by dividing particle displacement by elapsed time for sequential images of the frames surrounding the impact. Constraining these restitution values provides an important physical parameter that is necessary for discrete element simulations [Malone and Xu, 2008].

We conducted a suite of experiments consisting of 24 individual flows. Three ramp inclinations, 10, 20, and 30 degrees, were employed for both the rigid and erodible bed sections. For the rigid bed case we used two focus areas: one was centered on the release gate, and a second was located downslope, and centered 0.185 m before the terminal end of the apparatus. Four flows were initiated at each inclination (two per focus area). After each run we measured and recorded run-out distance at the terminus, evacuated distance from initiation point, and jamming height at the head and toe of the flow deposit (Figure 4). Run-out distance is the horizontal distance the flow reached beyond the apparatus walls and into the collection hopper. Evacuated length is the downslope distance between the flow deposit and the retaining wall at the top of the flow initiation point. Jamming height at the head and toe describes the height of the flow deposit at the retaining wall located above the flow initiation point and the terminal end of the retention hopper, respectively. For flows exhibiting evacuated length, no jamming occurs at the head of the deposit. Accordingly, for flows exhibiting run-out distance less than 0.30 m, no jamming occurs at the toe of the deposit. The high-speed imagery captured during the experimental runs provided a means of recording flow duration, bed forces, chain counts, chain lengths, chain frequency, and fringe orders (force magnitudes). For the erodible bed case we also used two focus areas: the first was the same as for the rigid bed experiments, and a second framed the erodible bed section downslope from the release gate. Four flows were run at each inclination for the erodible bed case, two for each focus.
unconfined, gravity-driven, dense granular flow. High-speed video recorded the experiment, and the final deposit was recorded with a still image.

[15] Still frame images from the high speed recordings of each incline – bed pairing were used to evaluate localized (force chain) versus averaged (from flow height) bed forces, and to determine the correlations between bed force orientations with chain inclinations relative to the substrate and total bed force magnitudes. Criteria for the image selections were clarity of force chains, fringe patterns and disk morphologies, as well as visibility of the granular flow surface in measurable proximity to the substrate.

2.2.1. Calibration

[16] In order to quantify the force magnitudes carried within the observed force chains, a proper calibration of the force-strain relationship was necessary. Conventional low elastic modulus PhotoStress calibration is achieved by an imposed-curvature method (Vishay Precision Group, Calibration of low-modulus PhotoStress coatings by the imposed-curvature method, 2010, http://www.vishaypg.com/docs/11238/_vmr-tc0.pdf). For most PhotoStress coated surfaces, the traditional approach to photoelastic calibration is to impose a series of known strains on simply shaped (typically a beam or rod) material with photoelastic coating, record the applied force and resultant fringing, and use this information to quantify the strain and/or force to the more complex structure under investigation. Multiple fringe cycles (a fringe order greater than one) indicate rather large forces relative to the elastic modulus of the photoelastic material. For fringe orders less than one, the affected, or optically active, area of the photoelastic material increases with increased strain. However, when the fringe order exceeds one, the area of birefringence no longer requires growth. Instead, the fringe pattern may cycle within the same area. This is especially true when the optically active area considered is the entire surface area of the investigated material.

[17] Our experimental granular systems possess unique characteristics, which allow us to justify an alternative calibration technique for our work. Most importantly, the relatively small-magnitude forces in our experiments never produce fringe orders greater than one. This is due to the small size and density of the disks, and the small scale of the granular systems employed. The discrete, monodisperse character of the disks along with the limited fringing observed means that the area of optical activity on a given disk is proportional to the force imposed on the boundaries of the disk. We use this relationship as a proxy for an incremental calibration scheme that correlates contact forces with area of optical activity.

[18] Four threshold fringe magnitudes that encompassed the spectrum observed during the experiments were established and systematically reproduced: zero – no visible fringing; contact (or quarter) – fringe initiation at the contact point on disk boundary; half – fringe propagation reaches midway through a disk; and full – fringe propagation covers the full diameter of a disk. Figure 5a shows a schematic diagram of the calibration setup. A 2D hopper, used as an alignment guide for the disks, was suspended 0.003 m above the surface of a digital scale. Four disks were placed on the scale in a double layer arrangement, and their cumulative weight recorded; this constituted the “zero fringe” case. The double layer allowed for clear visibility of fringing, which is necessary for imaging and accurate reproduction of fringe

Figure 5. (a) Schematic view of the calibration setup used during our experimentation. An acrylic arm applies force to a photoelastic disk until the desired fringe magnitude is reached. The underlying digital scale records the weight change; which is used to calculate the applied force. (b) Plot shows the calibration results in terms of fringe threshold values. Since the fringing observed during experimentation falls within the bounds of our calibration data, interpolation between the discrete threshold values is justifiable. Note that the mean data points and deviation bars are offset so they are distinguishable from standard data.
magnitudes. A dSLR camera was used to image the calibration sequences. A manually controlled acrylic arm applied force to the upper disk until the appropriate amount of fringing was produced, and the scale weight reading was recorded. This method was repeated for each of the threshold fringe magnitudes, ten times each. The scale-weight readings were converted to force units that were normalized for an individual disk experiencing force applied at one point of contact by dividing the zero-fringe value by the number of disks used in the calibration setup. Subtracting the residual, self-applied force caused by the disk weight was the final refinement of the calibration data. Figure 5b shows the calibration results. The calibration data, although discrete to threshold values, encompass the magnitudes of fringing observed throughout the suite of our experiments. Owing to this fringe containment and the small optically active area (relative to the facial area of a disk) available between threshold fringe levels, a simple linear interpolation between calibration data points is reasonable. We define a quantity, the fringe factor, which is the fractional area of a disk that is optically active; for example, if a disk has a fringe factor of 0.5, then half of its facial area is optically active. Table 1 compares calibration fringe thresholds to the number of self-weighted disks required to generate a force equivalent to each fringe threshold.

### 2.2.2. Image Processing

Information from the calibration-sequence images was implemented into an image-processing algorithm that we developed using MATLAB’s image analysis tools to extract data from the experiments. Individual frames from the video clips of each run were parsed into sequential tiff images. The image sequences were processed through the program, which identified the force chain fringe patterns and converted them into binary form, filtered the magnitudes by fringe thresholds, and computed several parameter values for the selected force chains. The program saved output files containing force chain counts, average and total chain lengths, and respective bed force magnitudes.

### 3. Results

Flow durations for each incline, recorded as the time elapsed between flow initiation and establishment of a static deposit, were: 10° slope, 1.74 ± 0.16 s; 20° slope, 2.18 ± 0.13 s; and 30° slope, 2.62 ± 0.21 s. Table 2 provides measured deposit values for several physical parameters depicted in Figure 4. Animation 1 and 2 show video examples of rigid bed and erodible bed experimental granular flows, respectively. \(^\text{[20]}\)

\(^\text{[21]}\) Bed force time series reveal irregular distributions of discrete localized bed forces for each incline in both time and magnitude. The bed-force time series are presented in Figure 6 for each inclination of the rigid bed experiments. The data, a summation of localized force chain) and averaged (flow height) bed forces, focus on the bed forces within an approximately 0.012 m length section on the ramp surface over the duration of each flow. These time series show bed forces at one position on the substrate as the flow passes. Visual inspection of the active flows also revealed irregular spatial force chain distribution patterns. \(^\text{[22]}\) Peak forces imparted by force chains on the bed greatly exceeded mean forces due to the flow thickness, demonstrated in Figure 7. The mean local bed force, the time-averaged value of the measured localized bed force magnitudes, was obtained from analyzing the fringe factor of the chain member contacting the substrate. The time-averaged flow height bed force was estimated from the

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**Table 1. Applied Force and Equitable Disk Weight in Terms of Calibrated Fringe Threshold Values**

<table>
<thead>
<tr>
<th>Fringe Threshold</th>
<th>Number of Disks to Equal Applied Force</th>
<th>Force Applied During Calibration (N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>None</td>
<td>1</td>
<td>0.001</td>
</tr>
<tr>
<td>Contact</td>
<td>9.55</td>
<td>0.009</td>
</tr>
<tr>
<td>Half</td>
<td>36.68</td>
<td>0.036</td>
</tr>
<tr>
<td>Full</td>
<td>70.34</td>
<td>0.068</td>
</tr>
</tbody>
</table>

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**Table 2. Dimensional Measured Values for Flow Deposits**

<table>
<thead>
<tr>
<th>Incline</th>
<th>10°</th>
<th>20°</th>
<th>30°</th>
</tr>
</thead>
<tbody>
<tr>
<td>Run-out Distance (m)</td>
<td>0.041 ± 0.028</td>
<td>0.162 ± 0.026</td>
<td>0.300 ± 0.000*</td>
</tr>
<tr>
<td>Evacuated Length (m)</td>
<td>0.000 ± 0.000</td>
<td>0.000 ± 0.000</td>
<td>0.056 ± 0.028</td>
</tr>
<tr>
<td>Jamming Height at Top (m)</td>
<td>0.186 ± 0.002</td>
<td>0.099 ± 0.003</td>
<td>0.000 ± 0.000</td>
</tr>
<tr>
<td>Jamming Height at Bottom (m)</td>
<td>0.000 ± 0.000</td>
<td>0.000 ± 0.000</td>
<td>0.009 ± 0.003</td>
</tr>
<tr>
<td>Jamming Height at Bottom (m)</td>
<td>0.000 ± 0.000</td>
<td>0.000 ± 0.000</td>
<td>0.002 ± 0.003</td>
</tr>
</tbody>
</table>

*The available run-out distance in the collection hopper was 0.30 m; therefore this represents a minimum value.

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**Figure 6.** Bed force time series describes cumulative bed force magnitudes due to force chains combined with the flow body for each inclination during rigid case runs. Red inverted triangles, blue squares, and green triangles represent the 10-degree, 20-degree, and 30-degree cases, respectively.

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1Animations are available in the HTML.
values include results from both focus areas of the rigid-bed and erodible-bed images, the localized force are significantly higher than the rigid case. On the basis of increase), the localized bed forces for the erodible section the flow height bed force values (the flow height does not force propagation. Because this extension is not reflected in section communicates with the flow body through extended flow-bed contact forces. The material in the erodible bed body during flow-substrate interaction, which enhances the erodible-bed show an elastic feedback response to the flow

Figure 7. Localized bed force magnitudes from force chains compared to averaged bed force magnitudes calculated from flow body height for rigid and erodible cases. The dashed line illustrates equity between force types, which emphasizes the significant force localization. Red inverted triangles, blue squares, and green triangles represent the 10-degree, 20-degree, and 30-degree cases, respectively. Solid markers indicate rigid bed data (denoted by “R” in the legend) while open-face markers indicate erodible bed data (denoted by “E” in the legend).

height of the flow normal to the force chain-substrate contact point to give a point of comparison with those forces we would predict from a depth-averaged perspective (i.e., to compare the results to depth-averaged continuum models as a base-line). Depth-averaged continuum models [Denlinger and Iverson, 2004; Gray et al., 1999; Vreugdenhil, 1994; Pudasaini et al., 2005] often calculate normal bed stresses by assuming that vertical stresses are dominated by weight of the flow immediately above the bed. In addition, Denlinger and Iverson [2004] proposed that a fluctuating vertical acceleration term is needed to account for the equivalent of downslope and centripetal acceleration for flows over complex three-dimensional topography. The flow height bed force we report here is an estimated value used to reference the order of magnitude we expect from depth-averaged continuum models for normal bed forces. Localized (force chain) bed force relative to flow height bed force magnitude was 435.62 ± 0.59% for rigid bed conditions and 738.32 ± 0.57% for erodible bed conditions. Error values represent one standard deviation for the cumulative data about the mean.

Photoelastic disks occupying the cutout area in the erodible-bed show an elastic feedback response to the flow body during flow-substrate interaction, which enhances the flow-bed contact forces. The material in the erodible bed section communicates with the flow body through extended force propagation. Because this extension is not reflected in the flow height bed force values (the flow height does not increase), the localized bed forces for the erodible section are significantly higher than the rigid case. On the basis of 29 rigid-bed and 14 erodible-bed images, the localized force values include results from both focus areas of the experiments. Although the precise value of the averaged localized force is a result of the particular location chosen for the measurements and the geometry of the apparatus, our results indicate that considerable bed force excursions due to force chain development are likely in an unconfined, gravity-driven granular flow.

We observed ejection of bed particles by the flow at each ramp inclination, with a positive correlation between the number of ejected particles and ramp inclination (higher inclines had more ejected bed particles). During the two separate flows at each ramp inclination, the total number of ejected bed particles was 1, 5, and 27 for the 10-degree, 20-degree, and 30-degree ramp inclinations, respectively. Each ejected particle was entrained into the body of the granular flow. Deposition of flow particles into the erodible bed section after bed particle ejection was also observed but not quantified.

Near steady state conditions for chain lengths are reached quickly relative to flow duration (after ~0.5 s the chain length values show little variance). Figures 8a and 8b presents time series showing 2 s durations of average length, total length, and number of force chains for each incline from rigid bed and erodible bed experiments, respectively. No distinguishable patterns were recognized upon analysis of the chain length and number regressions. Because the area captured by the camera frame was limited, data presented in the time series plots are representative of a fraction of the flow body.

Using the Pearson product-moment method [Dutilleul et al., 2000; Trauth, 2007], no significant correlation was found between force chain inclinations and the components (normal and shear) of the bed forces. Although no strong correlations were discovered, the chain inclinations and total bed force magnitudes showed small positive correlations; whereas the normal versus shear components displayed a medium negatively correlated relationship. Chain inclinations describe the force chain angle proximal to and relative to the flow substrate. Trauth [2007] provided a detailed description of the Pearson product-moment method, which measures the strength of linear dependence between two variables.

Because photoelastic material was used for the erodible-bed section, our experiments provided an avenue to investigate bed force propagation into the substrate. Employing the same techniques used to determine bed forces at the flow-substrate interface, we recorded the extent of birefringence observed in the cutout bed section resulting from flow contact with the substrate. Propagation extent versus bed-force magnitude is plotted in Figure 9. Propagation was limited by edge effects of the section domain so that the represented values are minimum values. The data for the bed-force propagation show no significant correlation between applied force and propagation extent, as the correlation coefficient of 0.328 is much lower than the 0.532 minimum value required for significance at the 95% confidence level at the given sample size. We find evidence of birefringence in the substrate ahead of the flow front due to forces applied upslope by the flow (Figure 10).

4. Discussion

Our experiments provide clear evidence of extreme localized (force chain) bed forces relative to average (from flow height) bed forces, considering “extreme” to be greater
by at least an order of magnitude. This work highlights that force chains, even in low-slope flows, can produce particle chain networks that may initiate stresses exceeding critical shear thresholds and may contribute to entrainment in granular systems. Traditional continuum models that derive averaged bed force values currently do not address the contributions of force chains to the flow substrate, and thus may under-predict bed forces locally. However, flow models that employ grain scales may be able to resolve these localized forces [Mangeney et al., 2007; Rattanadit et al., 2009; Reddy and Kumaran, 2010; Rycroft et al., 2009].

[29] Force chain processes at the beds of granular flows likely contribute to the mechanism of entrainment through either physically ejecting grains due to the localized forces, as seen in our experiments, or by modifying bed conditions. Schürch et al. [2011] used a combination of estimated maximum flow depths with elevation changes to evaluate the effect of flow depth on the probability of erosion in debris flows. The work shows that substantial erosion is more likely with increased flow depth, but also that a wide range of outcomes is possible at any given flow depth. For most debris flows, flow depth is largest at the front of the

Figure 8. (a) Time series of average length, total length, and number of force chains for each incline from rigid bed experiments. (b) Time series of average length, total length, and number of force chains for each incline from erodible bed experiments. Red diamonds, blue squares, and green diamonds indicate 10-degree, 20-degree, and 30-degree ramp inclinations, respectively.
flow, and flow depth influences forces acting on the channel bed by way of three mechanisms: (1) increased basal shear stresses, (2) the impact stresses of coarse particles in the flow front, and (3) hydraulic pressure at the flow front [Schürch et al., 2011]. Iverson et al. [2011] observed a positive correlation between water content of the substrate and scour depth for large scale experimental debris flows, and Schürch et al. [2011] noted that coarse debris flow fronts have very low fluid pressures. The influences on the bed physics of fluid interactions within granular flows are beyond the scope of this work, however low pore pressures at flow fronts may allow granular processes such as force chains to operate more effectively. Although not evaluated in these experiments, the propagation of force chains into the subsurface below and in front of the flow may also aid entrainment by modifying pore pressures in those substrates that are at or near liquid saturation.

[30] If a force chain is oriented obliquely to the bed it contacts, which was commonly observed in our experiments, then the force transmitted to the substrate by that chain contains normal and shear components. Interestingly, in our experiments the shear bed force components derived from force chains carried the same order of magnitude as the normal bed force components for the chain inclinations observed. This further supports the assertion that force chains in granular flows may contribute to the shearing mechanism necessary to “pluck” substrate material, initiate entrainment, and potentially alter flow momentum. This is not to say, however, that force chain dynamics are the only mechanism contributing to substrate entrainment in natural granular flows.

[31] Similarity in threshold behavior for substrate entrainment for unrelated systems occurs because entrainment thresholds depend on entrainment inducing bed forces themselves, and not on the agent producing the forces. Granular flow and fluvial flow systems may both exhibit critical threshold phenomena for substrate entrainment [Ancey et al., 2008; Arulanandan and Perry, 1983; Garcia and Parker, 1991; McDougall and Hungr, 2005; Sarmiento and Falcon, 2006; Zanuttigh and Lamberti, 2007]. It is unclear whether the basal shear stresses in granular flows are analogous to a critical shear stress for fluvial or other fluid entrainment processes, as other effects such as grain impacts or water content are likely relevant [Schürch et al., 2011; Iverson et al., 2011]. Previous work showed the similarities between granular and fluvial bed load systems [Frey and Church, 2011; McDougall and Hungr, 2005], suggesting that comparable physics may dominate each of these regimes. Force chains may supply a viable mechanism capable of producing shear forces necessary for entrainment in granular systems.

[32] Our experiments also indicate significant propagation of bed forces into the flow substrate, which is also a result of dynamic force chain processes. Although limited due to edge effects of the experimental apparatus domain, our data suggest that bed forces applied during force chain contact with the bed can propagate on the same length scales as force chains within the flow body. It is important to note that the limited domain size in our erodible bed experiments resulted in near-crystalline particle arrangements in the bed. Examination of static granular systems has shown that increases in disorder correlate to decreases in force propagation [Geng et al., 2003]. However, Geng et al. [2003] also reported that long-range force correlation for anisotropies in granular systems are caused by shearing motion. For force chains contacting the bed near the flow front, our experiments demonstrate that the bed forces may propagate ahead of the flow front and affect the substrate particles before the flow body reaches their proximity. This may potentially influence pore pressures in front of the flow in the case of wet substrates. Because force chains can cause significant perturbations to the forces at the bed, this work suggests that sub-grid models (varied spatial resolution) may be necessary when applying continuum models to granular flow erosion problems. Furthermore, our results imply that discrete-element computations may be particularly useful in examining force chain processes in geophysical flows [Schwaiger and Higman, 2007; Aharonov and Sparks, 2004].

[33] The low contact stiffness of the material we employed in our experiments relative to natural material raises questions concerning the applicability of our results to natural systems in terms of force chain stability due to an expected reduction in contact times for stiffer particles. Campbell [2003] showed that force chain stability in confined systems is dependent on confining pressure, restitution coefficient, and chain length. From this description, force chain stability in confined systems results from a balance of confining pressure and loading on the constituent particles within the chain. With the approach of Campbell [2003] force chains with “unloaded” contacts are unstable because one end of the chain is unconfined.

[34] However, force transmission via force chains does not require that the chains remain stable for long periods of time. Indeed, our experiments reveal significant force propagation through chains that persist for at most the duration of one high-speed video frame (1/200 s). Because our temporal resolution is 1/200 s, we cannot definitively say that force chains captured in our images persist even for that long. Our
data clearly indicate that force chains provide localized forces to the substrate, but we can use the granular time scales presented by Sun et al. [2010] to better illustrate why this occurs. Three granular time scales are defined: $t_m$ – microscopic time scale, $t_c$ – macroscopic time scale, and $t_R$ – Rayleigh (mesoscopic) time scale. The microscopic time scale

$$ t_m = \frac{d}{\sqrt{P_r}} \quad (1) $$

denotes the time for particle displacement (with density ($\rho$)) over the distance of a particle diameter ($d$) subjected to local pressure ($P$), which is the typical time scale of particle rearrangement. The macroscopic time scale

$$ t_c = \frac{1}{\gamma} \quad (2) $$

is linked to the contact lifetimes during the particle rearrangements occurring during a flow with shear rate ($\gamma$). The Rayleigh (mesoscopic) time scale

$$ t_R = \frac{\pi d}{0.163 \nu + 0.877 \sqrt{\rho G}} \quad (3) $$

is the time for a Rayleigh wave to propagate along a particle surface with Poisson’s ratio ($\nu$) and shear modulus ($G$).

**Table 3.** Parameter Values Used for Granular Time Scale Calculations

<table>
<thead>
<tr>
<th>Time Scale Parameter</th>
<th>$t_m = 6.52 \times 10^{-4}$ s</th>
<th>$t_c = 6.25 \times 10^{-2}$ s</th>
<th>$t_R = 5.85 \times 10^{-4}$ s</th>
</tr>
</thead>
<tbody>
<tr>
<td>d</td>
<td>0.006</td>
<td>0.006</td>
<td>0.006</td>
</tr>
<tr>
<td>$\rho$</td>
<td>1180</td>
<td>1180</td>
<td>1180</td>
</tr>
<tr>
<td>P</td>
<td>1000</td>
<td>16</td>
<td>$1.3 \times 10^8$</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>16</td>
<td>16</td>
<td>0.5</td>
</tr>
<tr>
<td>Units</td>
<td>m</td>
<td>kg m$^{-3}$</td>
<td>Pa</td>
</tr>
<tr>
<td></td>
<td>kg m$^{-3}$</td>
<td>s$^{-1}$</td>
<td>Pa</td>
</tr>
<tr>
<td></td>
<td>m</td>
<td>kg m$^{-3}$</td>
<td>n/a</td>
</tr>
</tbody>
</table>
The mesoscopic time scale represents the time for an elastic wave to propagate through a particle contact, i.e., force propagation.

[35] Calculating these characteristic timescales for the disks in our experiments, and assuming a force chain consisting of 10 disks we find: \( t_m = 6.52 \times 10^{-3} \) s, \( t_c = 6.25 \times 10^{-3} \) s, and \( 10^3 t_{fr} = 5.85 \times 10^{-3} \) s. Table 3 presents the parameter values used in the time scale calculations. Force propagation occurs over a shorter time period than the lifetime of particle contact or particle rearrangement, which indicates that the force chains effectively instantaneously transmit forces relative to other flow processes. This also illustrates that contact times do not scale with particle stiffness in dense granular flows, alternative to the trends observed in systems dominated by binary collisions.

5. Conclusions

[36] Photoelastic experiments provide quantitative evidence of dynamic force chain activity in gravity-driven granular flows, and reveal the significance force chain activity carries for conditions at the flow substrate. This work demonstrates that force chains, regardless of their stability, transmit high magnitude localized forces to the substrate of dense granular flows. Our experiments provide a data set to validate discrete-element granular flow models both qualitatively and quantitatively. Inter-particle force and bed-force data provide constraints on force magnitudes, and images obtained from the experiments reveal geometric information in both spatial and temporal domains. Future experiments of this type may increase scale sizes for the experimental systems, which would be especially useful for the erodible bed case. A larger erodible bed section may eliminate the observed edge effects and allow for more amorphous substrate particle arrangements. Polydispersity within the granular media and addition of fluids into the system are planned to strengthen experimental applicability to natural systems.

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