Aspects of a Discontinuous Galerkin Approach for 3D Dynamic Rupture Modeling in the Case of a Complex Fault System

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Motivation

Possible applications:

- Understanding earthquake source physics: initiation, propagation, and restarting effects
- Ground motion prediction
- Hazard assessments
- Seismic risk

Why with the Discontinuous Galerkin approach?

- High-accurate results of the rupture process – no spurious oscillations in the spectra
- Enables use of unstructured meshes – curved or kinked faults, branching, surface rupture, fault interaction
- High-accurate simulation of the wave propagation including heterogeneous media and topography
- Excellent scalability - large scale simulations
Discontinuous Galerkin Approach

Numerical approximation of the solution:

\[
\left( Q_h^{(m)} \right)_p (\xi, \eta, t) = \hat{Q}_p^{(m)}(t) \Phi_l(\xi, \eta)
\]

- \( \Phi_l \) are orthogonal basis functions
- diagonal mass matrix

Integrating the governing equations in space and time in the Discontinuous Galerkin (DG) framework gives

\[
\int_t^{t+\Delta t} \int_{\mathcal{T}(m)} \Phi_k \frac{\partial Q_p}{\partial t} \, dV \, dt + \sum_{j=1}^{3} F_p^{j} - \int_t^{t+\Delta t} \int_{\mathcal{T}(m)} \left( \frac{\partial \Phi_k}{\partial x} A_{pq} + \frac{\partial \Phi_k}{\partial y} B_{pq} \right) Q_q \, dV \, dt = 0
\]

where the numerical flux is given by

\[
F_p^{jk} = A_{pr} \int_t^{t+\Delta t} \int_{S_p} \Phi_k \tilde{Q}_r \, dS \, dt
\]
Dynamic Rupture within the Discontinuous Galerkin Approach

Treat dynamic rupture as a **boundary condition** using the flux term:

- flux provides information at element interface
- solve friction law
- in case of slip, impose traction and fault parallel velocities
- solve inverse Riemann problem

\[ Q_j^- \quad Q_j^+ \]
\[ x_n=0 \]

(de la Puente et al., 2009)
Verification – TPV3 SCEC Test Case

(Harris et al., 2004)

- spontaneous rupture propagation on a straight fault
- homogeneous fullspace
- linear slip weakening friction

Comparison between
ADER-DG method order 4 and 200m triangles at the fault (larger tetrahedrons in bulk)
and
DFM - Finite Difference staggered-grid split node order 2 with 50m grid interval
and
MDSBI - Multidimensional spectral boundary integral with 50m grid interval
Verification – TPV3 SCEC Test Case

Verification – TPV3 SCEC Test Case

DFM data provided by Luis Dalguer. MDSBI data computed with the code of E. Dunham (version 3.9.10).
Verification – TPV3 SCEC Test Case

DFM data provided by Luis Dalguer. MDSBI data computed with the code of E. Dunham (version 3.9.10).
ADER-DG does not show spurious oscillations at the high frequency content of the seismograms!
Convergence Test for TPV3

Rupture arrival time is measured at 400 randomly distributed positions

<table>
<thead>
<tr>
<th>Method</th>
<th>Rupture time</th>
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</thead>
<tbody>
<tr>
<td>ADER-DG O2</td>
<td>3.28</td>
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<tr>
<td>ADER-DG O3</td>
<td>2.84</td>
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<tr>
<td>ADER-DG O4</td>
<td>2.83</td>
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<tr>
<td>ADER-DG O5</td>
<td>2.83</td>
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<tr>
<td>DFM^a</td>
<td>2.96</td>
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<tr>
<td>BI^a</td>
<td>2.74</td>
</tr>
</tbody>
</table>

^aDay, et al. (2005)
Application to the Landers Earthquake Fault System

California’s
San Andreas Fault

Landers 1992
M=7.3

San Andreas Fault

Map copyright © 2006 David K. Lynch

www.data.scce.org/faults
Application to the Landers Earthquake Fault System

www.data.scec.org/faults
Application to the Landers Earthquake Fault System

Strong mesh coarsening applied
Strong mesh coarsening applied

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Summary

- New approach of implementing dynamic rupture via fluxes (J. de la Puente '09)
- Verification with the SCEC test (TPV3, TPV5)
- Bimaterial applications under Prakash-Clifton regularization (not presented)
- Application to complex fault structures with branches (1992 Landers)
- Method should allow surface rupture, fault branching, curved and kinked faults
- No spurious high-frequency contributions in the slip rate spectra

Open Problems and Outlook:

- Observation of a non-zero normal stress when using highly unsymmetric mesh
- Testing of smooth heterogeneous initial stress distributions (and their implementation)
- More advanced friction laws: rate-and-state